A GAMMA-RAY COINCIDENCE EXPERIMENT USING COBALT-60

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ABSTRACT

An angular correlation experiment was carried out using cobalt-60. NaI (Tl) scintillation counters were used. The angle between the detectors was varied between 90°and 180°. At each angle coincidences were counted for thirty minutes. The resolving time was set at 30ns. Three sets of data were taken on three separate days. The angular correlation formula for the coincidence rate is a polynomial in $\cos^2\theta$ of the form:

$$W(\theta) = F\left[1 + K_2 \cos^2\theta + K_4 \cos^4\theta\right]$$

The aim of the experiment was to find the source strength and the coefficients K_2 and K_4. Here are the results:

DATE	10/2/98	12/2/98	17/2/98
S / mCi	1.47 ± 0.01	1.44 ± 0.01	1.43 ± 0.01
K_2	0.11 ± 0.12	0.063 ± 0.048	0.007 ± 0.05
K_4	0.041 ± 0.089	0.078 ± 0.090	0.074 ± 0.08

According to the laboratory information the source strength was actually (1.28 ± 0.01)μCi. Theory tells us that the value of K_2 should be 1/8 and K_4 should be 1/24. It is believed that the reason why the second and third sets of data agree so poorly with theory is because the threshold energy for incoming photons was not set correctly.

1. INTRODUCTION

Cobalt-60 is radioactive isotope of cobalt that decays by β-decay to nickel-60 with a half–life of 1.66×10^9s [2]. The nickel atoms that result from this decay are in an excited state and almost instantaneously emit γ-rays and fall into the ground state. Figure 1 summarises the decay scheme. The vast majority of the cobalt atoms decay to nickel atoms in the 4^+ state. These atoms emit two photons, γ_1 (1.17 MeV) and γ_2 (1.33 MeV). The first photon is equally likely to be emitted in any direction*. The direction in which the second photon is emitted however depends on the direction in which the first was emitted. If the angle at which the first is emitted is designated to be at 0°, the distribution of the angles at which the second photons are emitted

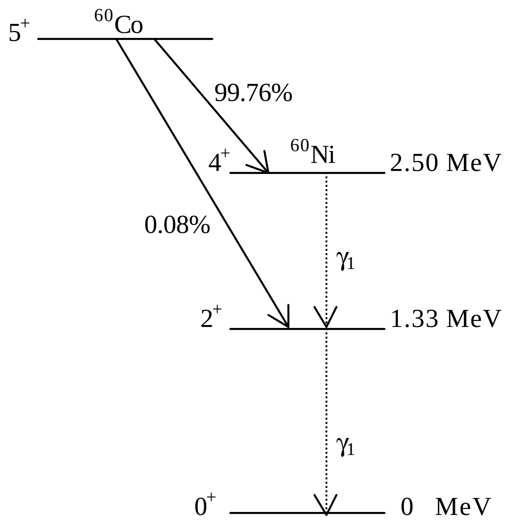

Figure 1. The decay scheme of ^{60}Co.

* This is not the case if in an external magnetic field, [1].

is represented by a polynomial in cosθ of the form

$$W(\theta) = F\left[1 + \sum_{n=1}^{L} K_{2n} \cos^{2n}\theta\right] \quad (1).$$

This is not a probability distribution per unit solid angle. The value of W is defined by this formula

$$W(\theta) = \frac{n_{co}}{n_1 n_2} \qquad (2)$$

where n_{co} is the corrected coincidence rate at a given angle (defined later) and n_1 and n_2 are the rates at which γ_1 and γ_2 respectively are emitted. The K values in equation (1) are constants depend on the total angular momentum and parity of the states concerned, [1]. F, is the related to the decay-rate of the source. L is the order of the multipole radiation in the first transition. In the case of cobalt-60 both transitions are electric quadrupole transitions (E2) and so L=2 and so equation (1) becomes

$$W(\theta) = F\left[1 + K_2 \cos^2\theta + K_4 \cos^4\theta\right] \quad (3)$$

The aim of this experiment was to find the values of the coefficients K_2 and K_4.

2. APPARATUS AND SET-UP

Figure 2 shows a schematic representation of the apparatus. A γ_1 is entering detector 1 and a γ_2 is entering detector 2. Whenever this happens it is called a coincidence. These are the events that are to be counted.

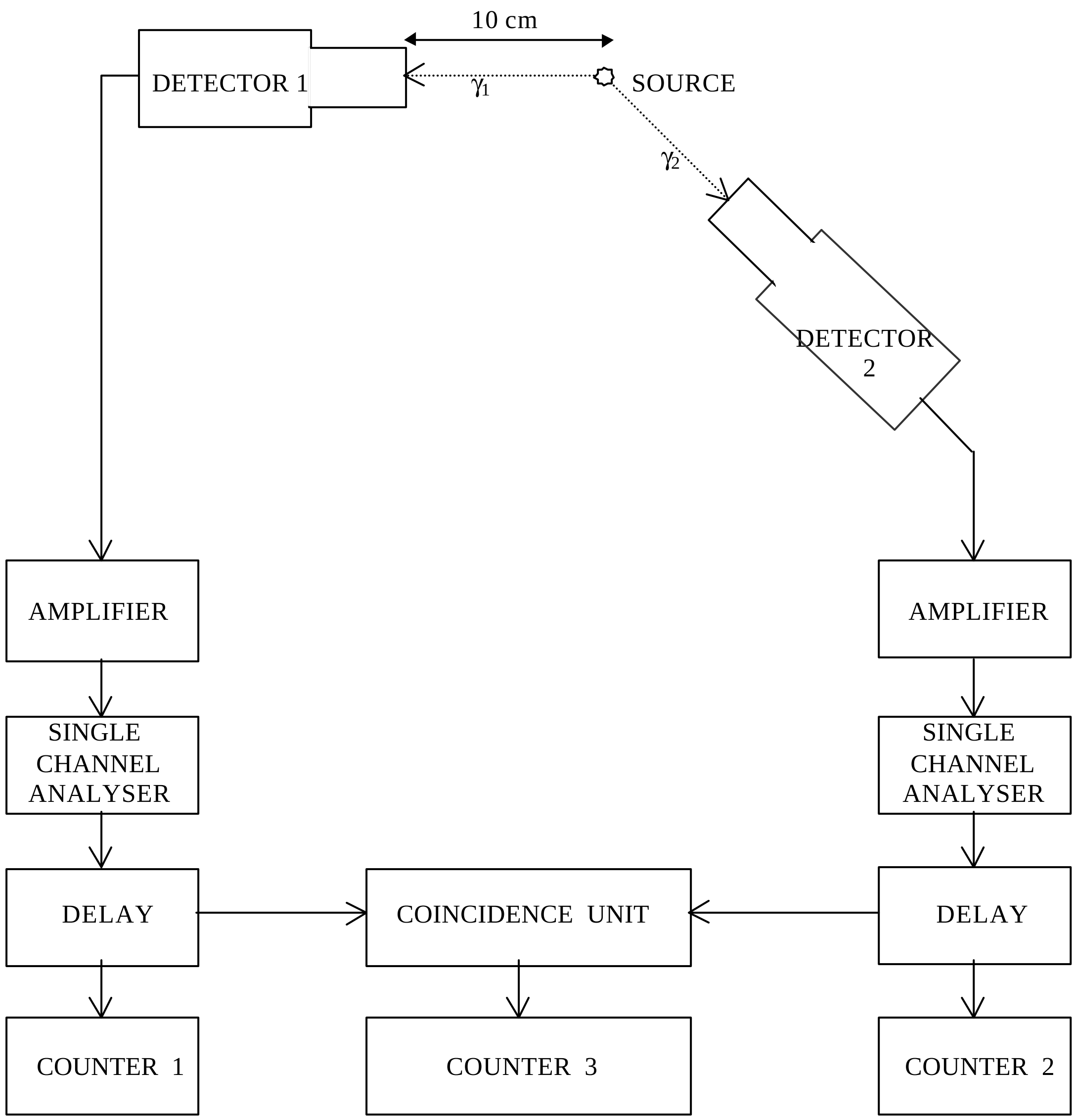

Figure 2. Schematic representation of apparatus.

The type detectors used were sodium iodide scintillation counters. At the end of the detector there is a large sodium iodide crystal which is doped with thallium, a semiconductor. A γ-ray photon, on entering the crystal forces some of the electrons within the ions to become excited. These electrons then fall back to their ground states and emit photons of visible light or close to the visible range. The probability of this occurring in sodium iodide crystal without the thallium is much lower because the thallium provides additional energy levels that are more easily reached by the electrons. On reaching a photocathode at the end of the crystal electrons are released and are drawn towards a succession of dynodes. The number of electrons is multiplied at each dynode and so eventually a measurable signal that is produced.

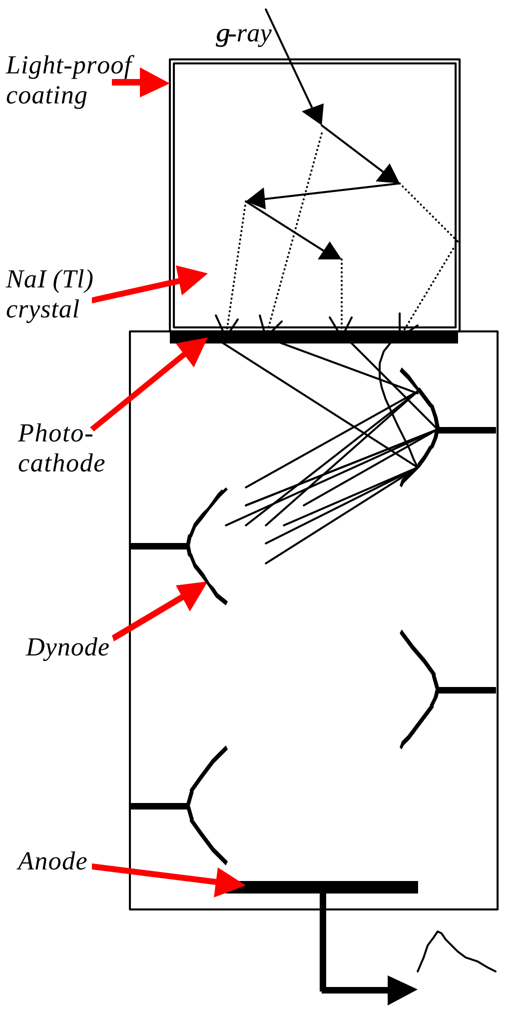

Figure 3. The scintillation counter and photomultiplier tube. The dotted arrows represent the paths of the lower frequency photons. The thin lines in the photomultiplier tube represent the path taken by the photoelectrons up the point where they're approaching dynode 2. Each dynode is set at a higher voltage than the one above The form of pulse that might appear on a CRO is shown at the bottom. [1], [2].

The output from the photomultiplier tube can be displayed as a set of extended pulses on a cathode ray oscilloscope. In our case the maximum height was ≈ 68mV and the maximum width was ≈ 90μs. Figure 4 shows what the amplifier did to the pulses in our experiment. All pulses whose maximum heights were below 2V were filtered out. This voltage corresponds to a γ-ray photon originally of energy 1.33 MeV that has been Compton scattered through an angle of 90° and thus out of the other detector, [1]. Approximately 370keV* of its energy is deposited in the detector. Photons of this energy or below must be discounted because they might enter the other counter and we do not want to detect any photons more than once.

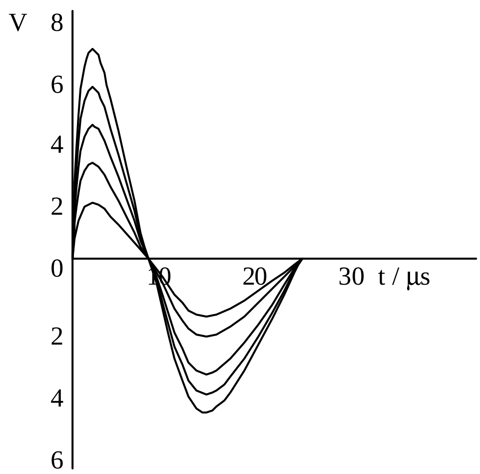

Figure 4. CRO trace after amplification.

The single channel analyser turned the set of traces into a single square pulse of depth 8V and width 0.6μs.

If the signals from each single channel analyser were fed straight into the coincidence unit it would highly unlikely that the experiment would work. This is because even if one photon were to enter one detector and another photon were to reach the other detector within the resolving time the signals would probably not reach the coincidence unit at the same time. To rectify this the "faster" signal has to be delayed so that they arrive at the same time. With the resolving time set at

* 2V / 7.2V × 1.33 MeV = 369 keV.

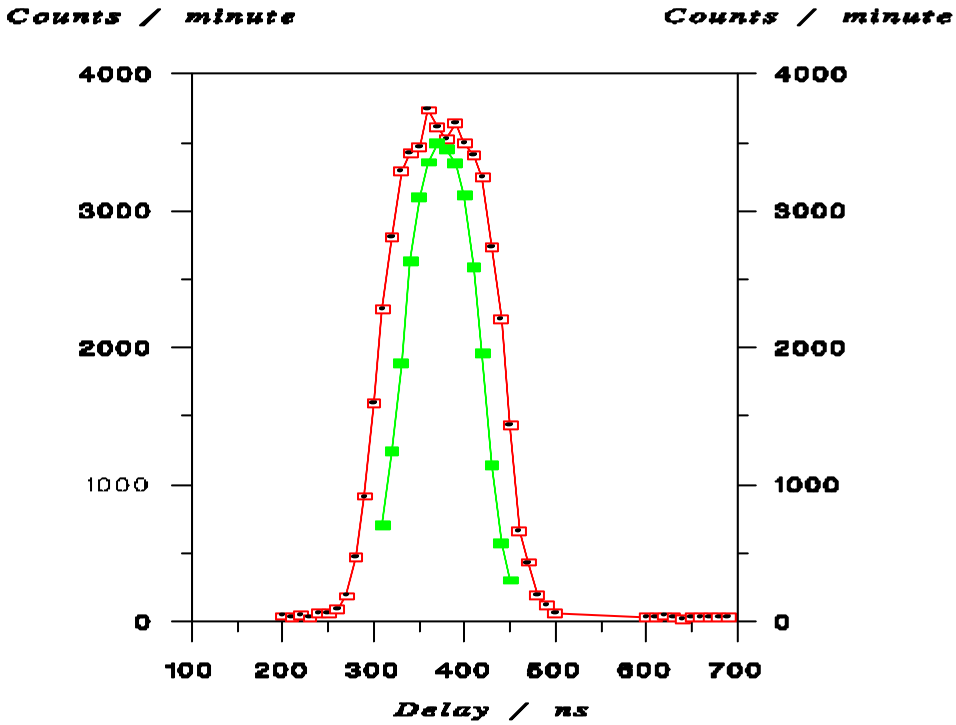

Figure 5. The number of coincidences counted in one minute plotted against the delay with the resolving time set at 30ns (red) and 10ns (green). The 20ns resolving time curve is not shown, but peaks at the same place.

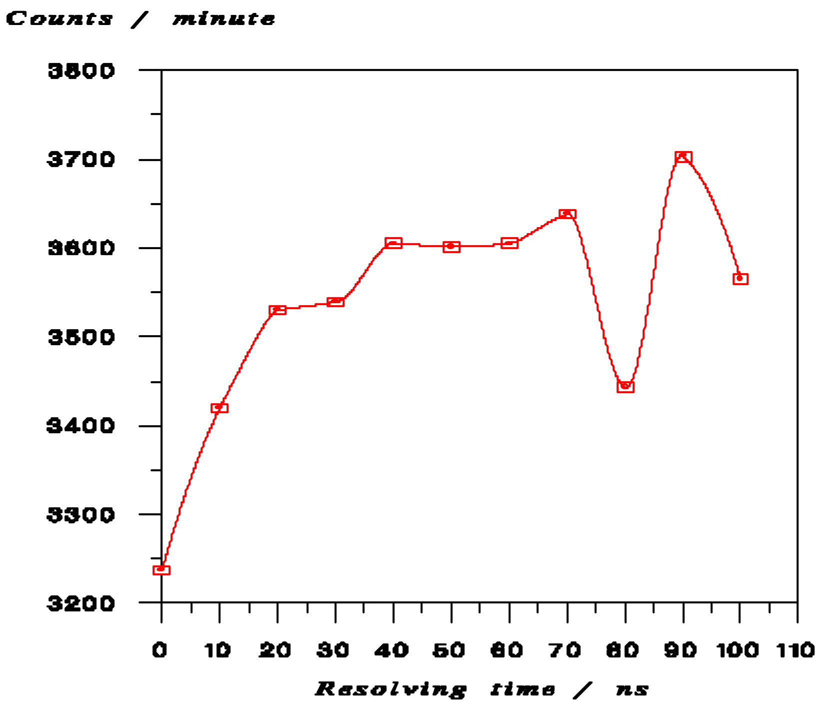

Figure 6. A graph showing how the number of coincidences counted changes in one minute when the resolving time is changed.

30ns some one-minute coincidence counts were made using delay times ranging from 0.2μs to 0.69μs. The angle between the detectors was 180° (i.e. they were facing each other). Further readings were taken using resolving times of 20ns and 10ns. Figure 5 shows that all three peaks lie close to 0.37μs. The fore-mentioned resolving time is the time within which γ_2 must arrive at the second detector once γ_1 has arrived at the first for the event to count as a coincidence. Using the delay time that we had established, more one-minute coincidence counts. The graph is shown in figure 6. At a resolving time of about 30ns the gradient of the graphline becomes much flatter. This is the resolving time we chose to use. If a smaller resolving were used some of the coincidences would be missed. At significantly higher resolving times the number of random coincidences increases beyond an acceptable level. A coincidence is a random coincidence when the two photons concerned were emitted by different nuclei.

3. EXPERIMENTATION

The 10^{th} of February 1998 was our first main data-taking day. With the detectors 180° apart the counters were switched on for 30 minutes. The readings on all three counters were noted. The same thing was repeated with
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angles 153.4°, 140.8°, 135.0°, 129.2°, 116.6° and 90°. Our reason for the choice of angles was that the squares of the cosine of the angles are 1, 0.8, 0.6, 0.5, 0.4, 0.2 and 0 respectively. Exactly the same experimentation was carried out on the 12^{th} and 17^{th} of February.

4. DATA CORRECTIONS

Before the coefficients could be calculated it was necessary first to subtract the random coincidences from the coincidence counts. Provided that the counting period is reasonably long (30 minutes is long enough) the number of random coincidences per second is given by this formula

$$n_r = 2n_1n_2\tau \quad (3), \quad [3]$$

where n_1 is the number of counts per second on counter 1, n_2 is the number of counts per second on counter 2 and τ is the resolving time in seconds.

Over a period of one week the decay rate of a ^{60}Co source falls by 0.3%. this however is not important because in calculating W it is necessary to divide the coincidence rate by the two count rates. One thing that has not been accounted for yet for is the fact that there are 0.08% more γ_2s emitted than there are γ_1s. The effects that this has on the coincidence rates though are negligible.

5. RESULTS

Displayed in table 1 below is the data collected on the 10^{th} of February. The data from the 12^{th} of February, without the counter readings, is displayed on table 2. The data from the 17^{th}, without the counter readings, is displayed in table 3. The errors are based on the principle that if N counts are collected in a given time, the error on that number is the square root of N.

Table 1: Data from 10/2/98

Angle / °	Counter 1 Reading	Counter 2 Reading	Counter 3 reading	Corrected coin. rate / s^{-1}	W / $10^{-6}s^{-1}$	Error on W /$10^{-6}s^{-1}$
90	733702	617836	2261	1.255	8.83	0.20
116.6	724856	615932	2229	1.238	8.84	0.20
129.2	719540	617524	2372	1.317	9.47	0.20
135.0	717919	618199	2377	1.320	9.50	0.20
140.8	718500	618654	2345	1.302	9.37	0.20
153.4	714255	619642	2408	1.337	9.65	0.20
180.0	715411	618469	2542	1.411	10.20	0.21

Table 2: Data from 12/2/98

Angle / °	W / $10^{-6}s^{-1}$	Error on W /10^{-6} s^{-1}
90	9.14	0.20
116.6	8.93	0.20
129.2	9.52	0.20
135.0	9.62	0.20
140.8	9.41	0.20
153.4	10.19	0.21
180.0	10.20	0.21

Table 3: Data from 17/2/98

Angle / °	W / $10^{-6}s^{-1}$	Error on W /10^{-6} s^{-1}
90	9.29	0.21
116.6	9.30	0.21
129.2	9.21	0.20
135.0	9.71	0.20
140.8	9.64	0.20
153.4	9.70	0.21
180.0	10.08	0.21

6. POLYNOMIAL FITTING

To fit a polynomial in $\cos^2\theta$ to the data a program known as "WLeg Fit" was used. It calculates a best-fit curve for the data used the associated Legendre polynomials. The ones of interest are:

$$P_0(\cos\theta) = 1$$

$$P_2(\cos\theta) = \frac{1}{2}\left(3\cos^2\theta - 1\right) \qquad [3]$$

$$P_4(\cos\theta) = \frac{1}{8}\left(35\cos^4\theta - 30\cos^2\theta + 3\right)$$

The correlation function, equation (2), can be rewritten as follows:

$$W(\theta) = F\left[1 + B_2Q_2P_2(\cos\theta) + B_4Q_4P_4(\cos\theta)\right] \quad (5)$$

B_2 and B_4 are constants Q_2 and Q_4 are attenuation coefficients which result from the fact that the detectors have a finite size. The crystals were 2" by 2" in size and they were 10cm away from the source. Q_2 was therefore 0.97 and Q_4 was 0.90, [4]. The program plots a graph, such as the one shown in figure 7, and calculates F and the products Q_2B_2 and Q_4B_4 for us. The coefficients K2 and K4 can be calculated from them.

The source strength is related to F by this formula,

$$S = \frac{1}{2F} \qquad (6)$$

It is only an approximation because it assumes that the efficiencies with which the two photons are detected are equal. The probability of higher energy photon passing straight through the crystal is higher than that of the lower energy photon. In the case of the lower energy photon though there is a greater chance that it might be Compton-scattered to an energy below the 370 keV limit. This is probably a reasonable assumption all in all therefore to assume that the two efficiencies are equal.

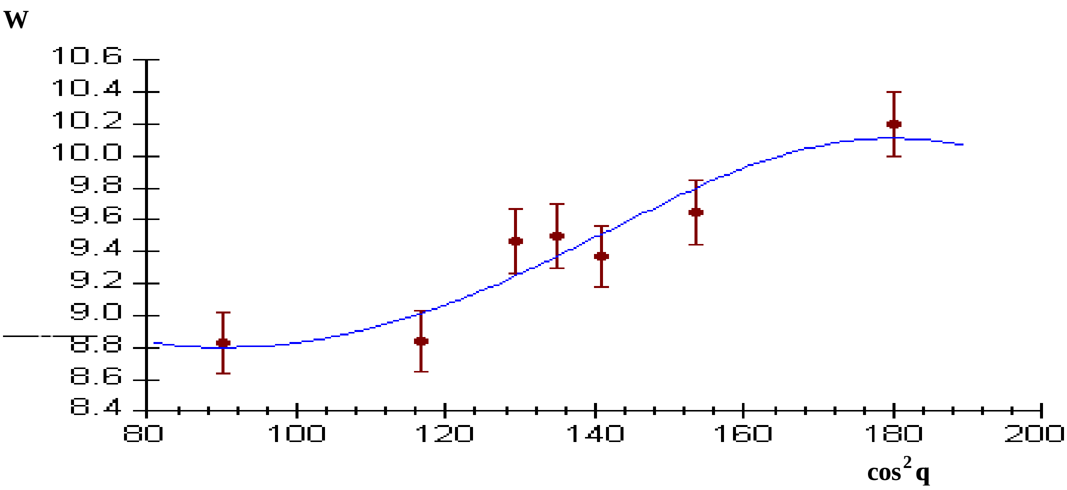

Figure 7. W(q) for the first set of data

Table 4 shows the final results

Table 4. Final Data

DATE	10/2/98	12/2/98	17/2/98
F	9.19 ± 0.08	9.37 ± 0.09	9.44 ± 0.09
S / mCi	1.47 ± 0.01	1.44 ± 0.01	1.43 ± 0.01
Q_2B_2	0.092 ± 0.019	0.085 ± 0.019	0.049 ± 0.019
Q_4B_4	0.008 ± 0.018	0.016 ± 0.019	0.017 ± 0.019
K_2	0.11 ± 0.12	0.063 ± 0.048	0.007 ± 0.050
K_4	0.041 ± 0.089	0.078 ± 0.090	0.074 ± 0.080

7. CONCLUSION.

The three estimated source strengths are all within 0.04μCi of each other, implying that 1.45μCi is quite a good estimate and that equation 6 is satisfactory. According to the data held within the laboratory the source used ought to have had a strength at that time of (1.28±0.01)μCi.

The Calculated K-values for $4^+ \rightarrow 2^+ \rightarrow 0^+$ transitions are exactly, [1],

$$K_2 = \tfrac{1}{8}\,(0.125)$$
$$K_4 = \tfrac{1}{24}\,(0.417).$$

The values obtained using the first set of data are close; but if we did not know what the angular momentum states in the decay were it might be difficult to work backwards deduce what they were from these values. In any case the errors are so large that it is difficult to trust the results. The values obtained using the other two sets of data are a long way off. The most likely reason for the discrepancy is that the photon energy threshold was not set correctly on those days. Making that mistake would significantly increase the number of random coincidences. This should have little effect on F, which is a scaling factor, but would squash the distribution vertically and cause the K-values to be smaller than they ought to be. The experiment has to be set up with great care.

8. REFERENCES

[1] Kenneth S. Krane, "Introductory Nuclear Physics" (2nd. ed. 1987, Wiley).

[2] Harald A. Enge, "Introduction to Nuclear Physics" (1966, Addison Wesley).

[3] Mary L. Boas, "Mathematical Methods for the Physical sciences" (2nd ed. 1983, Wiley).

[4] Ed Kai Seighbahn, "Alpha, Beta and Gamma Ray Spectroscopy" , volume 2 ,Table 3, appendix 9. (1st ed. 1966).