THE MACH-ZEHNDER FIBRE OPTICS INTERFEROMETER

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Mphys. Project Report
Experimentation from 22/9/98 until 18/12/98
Report submitted:27/ 1/ 99.

ABSTRACT

The aim of this experiment was quite undefined originally, we were given free range to investigate the phenomenon of temperature change or pressure change with the equipment available. The experiment itself was designed to allow the students undertaking it, a certain amount of choice in the way in which they pursued their study. We initially investigated temperature change (to avoid damage to the fibre) with a variety of methods and counting devices, but it soon became clear that with the equipment available, we could not produce results of a satisfactory standard, particularly considering the size of the errors we were observing. What we chose to do was instead to invest our time into the production of a new piece of equipment that would make our task easier, and reduce the errors we found significantly. However it soon became apparent that this was a very difficult job, as many hours were spent simply on the design of the hardware. It was our intention to complete the new hardware and then take fresh readings with it, to show the improvement in the results. However the time constraint has prevented us from completing the production of our hardware device at the time of writing this report, I am however confident that it will be completed and be available to the next students who undertake the experiment.

1. INTRODUCTION

1.1 Interferometry.

Whenever one wavetrain of electromagnetic radiation is superimposed over another the two interfere with each other. If both wavetrains happen to be both sinusoidal and coherent the result might be constructive interference, destructive interference or something in the middle depending on the phase difference of the two wavetrains. In this experiment a Mach-Zehnder fibre optic interferometer was constructed. As will be discussed later, such an interferometer can be used to measure several physical quantities, though in practice some of these are easier to measure accurately than others.

The reader will probably be familiar with the Michelson interferometer, shown in figure 1

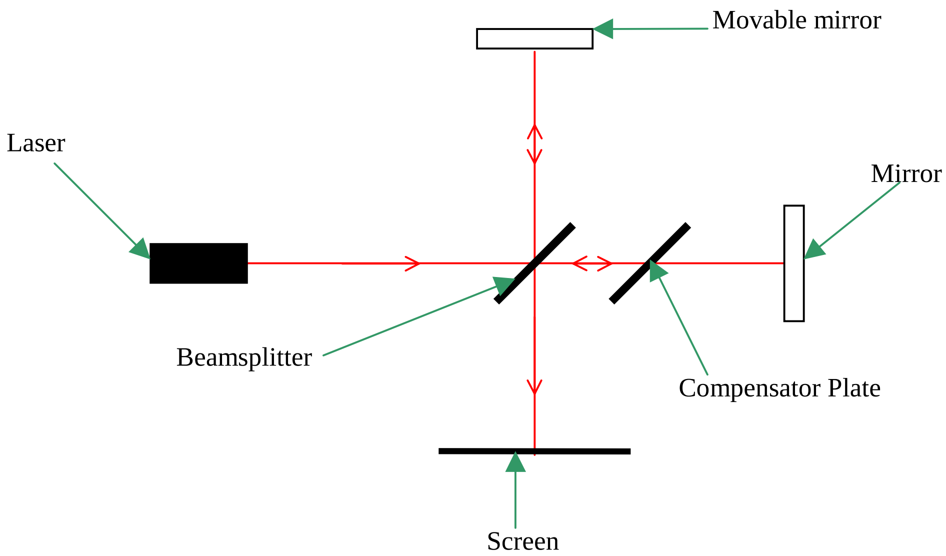

Figure 1. The Michelson interferometer.

If the laser is monochromatic a set of concentric circular fringes appear op the screen. The brightest fringe is either at or near the centre. The outer fringes appear narrower and dimmer. These fringes are known as fringes of equal inclination or Haidinger fringes, [1]. The exact pattern depends on the position of the moveable movable mirror. If the laser light is of the form A exp($i\omega t$), where A is the amplitude and ω is the angular frequency, the intensity at the centre of the fringe pattern is:

$$2A^2(1+\cos\phi) \qquad (1)$$

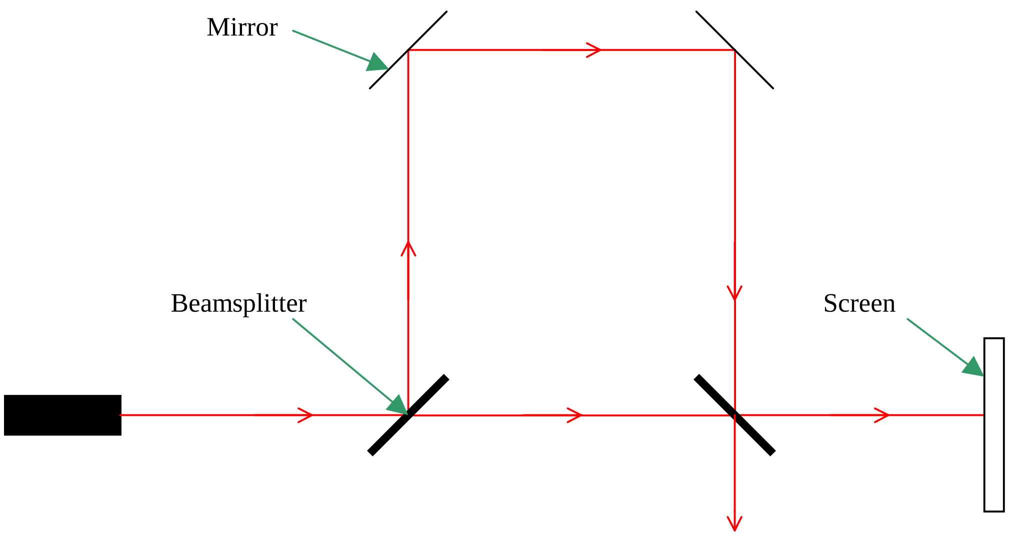

Figure 2. The Mach-Zehnder Interferometer.

where ϕ is the phase difference between the two paths. Another type of interferometer, which is very similar to the Michelson, is the Mach-Zehnder interferometer, [2]. One is shown schematically in figure 2. Such an interferometer is often used for measuring the densities of gases. A change in refractive index in one of the optical paths produces an effective path difference. The Mach-Zehnder interferometer does in fact produce two patterns. There is a phase difference of 180° between the two patterns.

1.2 Fibre Optic Cables.

An optical fibre is basically an optical waveguide. There are many types of optical fibre. The ones used in this experiment were single mode step index fibres. A step index fibre has a core wholly of one refractive index, surrounded by a cladding of a lower refractive index. For reasons of protection is surrounded by a jacket. The jacket is made of UV-cured dual acrylate , [3], which is a highly flexible thermosetting plastic, [4]. Without it the fibre would be very fragile. A step index fibre has a quantity called the numerical aperture, [5],

$$NA = \sqrt{n_1^{\,2} - n_2^{\,2}} \qquad (1)$$

where n_1 is the core’s refractive index and n_2 is that of the cladding. The acceptance angle of the fibre is

$$\alpha = \sin^{-1}\left(\frac{NA}{n_0}\right) \qquad (2)$$

where n_0 is the refractive index of the medium outside the fibre, which for air is close to unity. Rays incident on the fibre’s end at larger angles will not be contained within the core, and thus not emerge at the other end.

The use of a single mode fibre is required, to avoid the overlapping of many modes of laser light that could propagate along the fibre. We require the TEM_{00} mode only to propagate, this is the ‘usual’ bright central fringe, laser mode. Propagation of many modes leads to a variation in intensity across parts of the beam cross-section, which have uniform intensity with the propagation of the TEM_{00} mode only. If more than one mode was propagating it would also reduce the coherence length of the laser light, and would then only allow interference between the beams, if the path difference between the two arms was extremely small, this could prove extremely frustrating experimentally. For a given wavelength all fibre optics have a function known as “V-number” which is defined by, [5],

$$V = \frac{2\pi a}{\lambda} NA \qquad (3)$$

where a is the core’s radius and λ is the wavelength. The V-number must be below 2.405 for a fibre to single mode for the chosen frequency

1.3 The Mach-Zehnder Fibre Optics interferometer

In the case of the ordinary Mach-Zehnder interferometer the laser light passes through the air. Alternatively we could make it so that the light travels through optical fibres. There are several possible arrangements of such an interferometer. The type of we constructed is shown in figure 3. It includes two beamsplitters. The first one is a SIFAM L-2S-63-B-50 coupler. It is 45mm long, 2.9mm in diameter and is supplied with three single mode step index fibres attached. The second is a cubic beamsplitter. It consists of two right-angled triangular prisms with a gap between them of about 1 micron. As the gap is so narrow the incident photons, on reaching the gap do not know whether to be internally reflected or to pass straight through. This principle is known as frustrated internal reflection, [6]. The cube is designed so that half of them are transmitted and the other half are the other half are reflected. As with the ordinary Mach-Zehnder interferometer, two patterns in antiphase are produced. If we could change the length of the sensing fibre while keeping the reference fibre at a constant length we could create a phase difference. The fringes on the pattern would move.

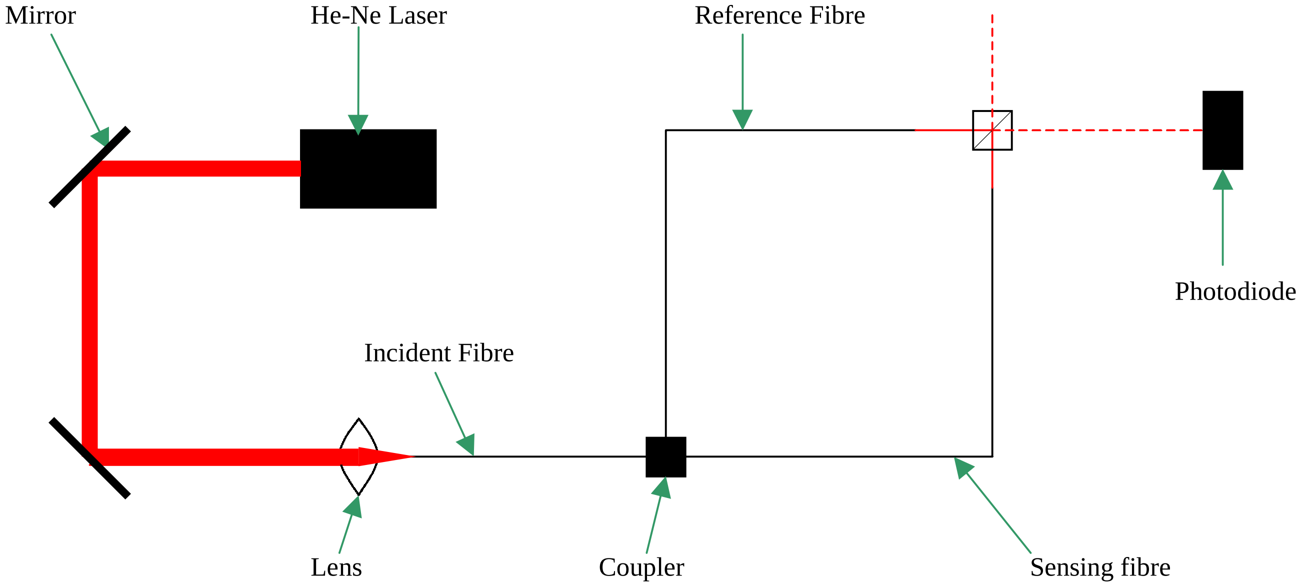

Figure 3. Our arrangement of the Mach-Zehnder fibre optics interferometer.

Suppose we were to apply a strain to the sensing fibre in the fashion shown in figure 4. We could measure the strain by measuring the angle of deflection, θ. Alternatively we could measure the strain by counting the number of fringes that would appear at the centre of the pattern and subsequently move outwards. Each fringe is equivalent to one wavelength of extension. One must remember that a wavelength in an optical fibre is shorter than a wavelength in air.

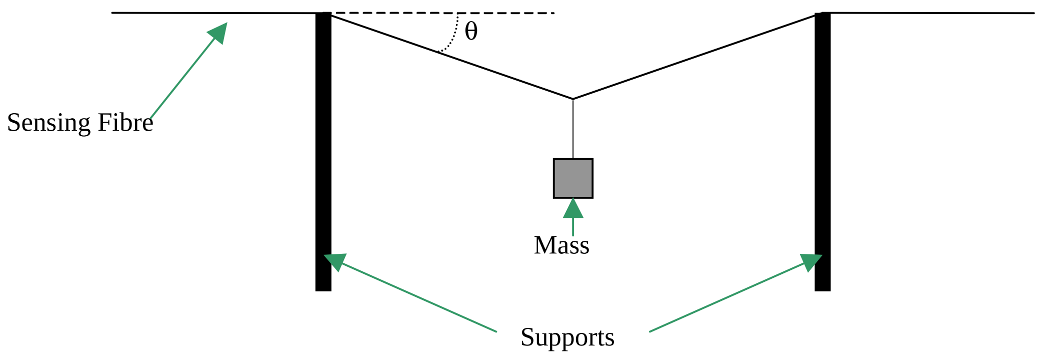

Figure 4. One method of applying a strain to the fibre

A less direct method of inducing a strain in the fibre would be to use a magnetostrictive material and apply a magnetic field. To do this the sensing fibre must be glued to a slab or cylinder of a magnetostrictive material, or alternatively a thin layer may be evaporated onto it, [7]. The equivalent method for inducing a strain using an electric field would be to attach part of the sensing fibre to a piezoelectric material. Another method of creating a path difference is by applying a pressure to a fibre

There is a means by which not only the length of the sensing fibre may be changed but also its refractive index. This can be done by changing the temperature of part of the fibre. Increasing the temperature increases fibre’s length (it also increases its width though we needn’t worry about that) and increases its refractive index. Both of these effects increase the path length of the sensing arm. The Mach-Zehnder interferometer could potentially be a very accurate thermometer. However for it to work most effectively the parts of the sensing fibre which is not in use must be kept at room temperature and so must the whole of the reference fibre. If they are not well insulated there will be unwanted fringe motions which limits the accuracy on measurements of not only temperature but other quantities as well. A helium-neon laser is ideal for any kind of measurements that are to made with an interferometer like this. This is because the natural line width of the reed 632.8nm line is just 2.0pm,

[8]. The coherence length of the laser in glass is ≈ 7cm, equivalent to 160 000 fringes. We are unlikely to want to measure extensions of that size.

2. PREPARATION OF INTERFEROMETER

2.1 Cleaving Fibres

Before the interferometer could be set up the ends of the coupler fibres had to be cleaved. The first step in cleaving each fibre end was to remove a short section of the jacket by dipping the end in xylene. They were then cleaved by scratching the cladding with a diamond blade and then pulling the end. The depth of the scratch determines whether the cleave will be good enough. It must not be too deep such that the core is damaged and also not so shallow such that the crack does not propagate at all and the cleave appears in a random position, [5]. If the cleave is not flat, that might be visible when inspected under a microscope. The best way of testing whether a cleave is good enough is to launch some laser light into the other end a look at the distribution of the light that emerges. For a single mode fibre the distribution should be circular. If not the distribution will probably be something resembling a comet (i.e. a circle with a wispy triangle sticking out). That end will have to be recleaved.

2.2 Measurement of the Numerical Aperture

The numerical aperture of a fibre can be measured by launching laser light into one end of the fibre from various angles and seeing whether any light emerges at the other end. The measured acceptance angle was 7.5° ± 0.5°. If we assume that the refractive index of air is 1, then from equation (2), NA = 0.13 ± 0.01, which is in rough accordance to SIFAM who state that NA = 0.11, [2]. The core's radius is 1.9µm, [3]. For a wavelength of 632.8nm the V-number 2.075, which is below 2.405 and thus the fibres are single mode. In spite of that there was some evidence in the form of half size fringe patterns that some amounts of other modes were propagating. There were also some very small patterns visible which were probably due to diffraction by dirt on the cubic beamsplitter.

2.3 Focusing the light into the fibre.

As figure 3 shows the laser light was focused into the end of the incident fibre using a converging lens. The focal length of the lens was 4mm. The beam diameter just in front of the lens was 2.0mm ± 0.2mm. It was not possible to focus all of the light into the fibre because 14° is larger than the acceptance angle.

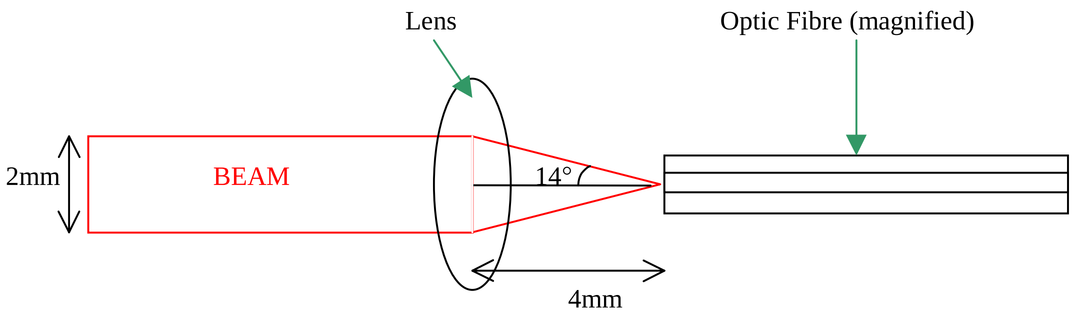

Figure 5. The beam is focused into the core of the fibre.

The maximum proportion of light that can enter is just 21.6%. The diameter of the core of the fibre is 3.8µm. This means that vertically the centre of the fibre's end must be within 1.9µm of the focal point for the maximum amount of light to be taken in. Horizontally it must within 7.6µm of the focal point. Because of this required accuracy level it is essential that the end of the incident fibre is held firmly in place and is isolated from external vibrations as much as possible.

In figure 3 there are two mirrors. These were required so that the laser beam would be at the correct height as to enter the lens apparatus properly. Sadly though some of the light is lost on reflection at the mirror surfaces. The power of the laser as measured with a metrologic radiometer was 0.90 ± 0.01mW. After the first mirror the power was reduced to 0.68 ± 0.01mW. After the second reflection it was reduced to 0.54 ± 0.01mW. The attenuation within the fibres is 12 dB/km, [3]. This is significant over long distances but over a distance of two metres the power is reduced by only 0.4%. The total output from the fibres should have been 0.12mW. The measured value was 62 ± 1 μW from each fibre, giving a total of 0.124 ± 0.002mW.

2.4 At the other end

At the other end of the interferometer the two fibre ends must be positioned correctly to within a similar level of accuracy such that the emerging beams interfere end produce a pattern like the one shown in figure 6 (appendix B). They must be at right angles to each other and must hit the interface of the beamsplitter cube at 45°.

2.5 Vibrational Isolation

Vibrational isolation is very important in an interferometer. This is principally important for keeping the fibre ends in the right places. It is also important for keeping the two mirrors at the right angles, [9]. The most significant laboratory noise mechanisms produce noise between 10 and 100 Hz, [10]. Electrical equipment for example is likely to produce noise at 50Hz. Figure 6 shows a photograph of our optical table. The table has a steel frame which is 72 cm high. The four legs of the table are embedded in pots of sand. On top the frame there is a 7.2cm a layer of sponge and above that a 5.6cm layer of marble. These two layers can be seen in figure 7 (appendix B). The resonant frequencies of these two materials is likely to be very different, thus cutting out a wide range of frequencies. On top of the lower breadboard there is a rubber ring and on top of that there is another optical tabletop.

2.6 Counting the fringes

When the interferometer was first set up, it was intended for the purpose of measuring large temperature changes. A photodiode was positioned 140cm away from the cubic beamsplitter, at the centre of the fringe pattern. Connected to that was a voltmeter which could set up so that it would give a single count each time the voltage rose above a certain level, but doing nothing when it fell below that level. In theory this simple set up could be used to count fringes as they move through a certain place in a certain direction when taking a measurement. It turned out that there were problems. These shall be discussed later.

3 TEMPERATURE MEASUREMENT

3.1 The effects of temperature on the path difference.

The change in phase due to a small change in temperature in a section of length L in the sensing fibre is given approximately by this formula, [5],

$$\Delta\phi = \frac{2\pi L n_1}{\lambda_0}\left(\frac{dn_1}{dT} + n_1\alpha\right)\Delta T \qquad (4)$$

where λ_0 is the light's wavelength in a vacuum, ΔT is the change in temperature and α the linear coefficient of thermal expansion. The core of the fibres was made of fused silica with a trace of germanium. It's refractive index for He-Ne light is 1.4616, [3]. Values for the rate of change of refractive index and the linear coefficient of thermal expansion at a temperature of 35°C were interpolated from tables in chapter 2 of Bansal and Doremus' "Handbook of Glass Properties", [11]:

$$\frac{dn}{dT} = 9.7\times10^{-6}\,K^{-1} \qquad \alpha = 4.23\times10^{-7}\,K^{-1}.$$

3.2 Initial attempts at measuring temperature changes.

Figure 8 shows how a coil of sensing fibre of length was immersed in a beaker of preheated water such that we could measure the phase changes that occur while the water cools. A thermocouple was used to give us temperature readings that the phase changes could be related. However the specific set up shown was not the one used initially. The length of the coils was 85.6cm. More importantly the reference fibre was encased by a rubber jacket for insulation but the sensing fibre was not encased at all.

It was clear from initial observations that staring at the fringe pattern and counting the fringes as they move was going to be an impossible task because for a lot of the time they moved too quickly. In any case having to stare at something keenly for a long time is not an enjoyable task.

Utilising the equipment available to us in the room we linked a battery operated photodiode and amplifier unit to the input of an electronic counter. The trigger level for the electronic counter was placed at a level, which would allow for the counting of each fringe as the intensity of the laser light pushed the photodiode output voltage past the trigger level. The setting of this trigger level to the optimum position was particularly tricky due to the sensitivity of the controls on the counter to even the smallest adjustment. The setting (or resetting) of the trigger level was repeated for each run of measurements, because it was found that after a break of just 1 hour (e.g. a lunch break), the trigger level would no longer activate the counter satisfactorily and it would have to be reset.

Often when taking a set of readings the counter would, almost at random, add several hundred counts to the total. This occurred during slow fringe movement, when the photodiode was detecting a transition from alight to a dark fringe. The cause of this was the photodiode output, which rose close to the trigger level but was held just below it, either by slow overall movement or by a temperature change halting the fringe movement. However the noise on the signal was sufficient to push the voltage over the trigger level, unfortunately for us the noise may do this several hundred times a second, adding to the total number of fringes counted and wrecking our results.

To avoid these problems we switched the recording device from electronic counter, to a chart recorder to graphically record the output from the photodiode. The idea behind this was that we could then analyse the photodiode output over a longer period of time and could come to a more considered decision upon which fringes were in fact moving in which directions, and what constituted a whole fringe movement in either direction. However it proved hard to constantly apply the same rules to define which fringes had ‘passed’ the sensor. The main problem with this method was the need to indicate on the chart paper the direction that the fringes were observed to be moving, and when the fringes were observed to change direction. This introduced a host of human response errors, from lapse of concentration to delay in signalling the direction change. With computer support to analyse the photodiode output this method would have been viable, but for us it was simply too error prone. We did make two efforts at measuring at measuring the number of fringes per °C temperature change:

Initial Temperature /°C	39.11	41.40
Final Temperature /°C	37.91	40.64
Forward Fringe Count	85	75
Backward Fringe Count	22	23
Net fringes per °C	52.5	69.7

Using equation (4) and the figures for 35°C (they should be approximately correct for these temperatures also) the fringe count per °C should have been 20.3. This suggests that the chart recorder method is not accurate enough.

3.3 Alternative Equipment for fringe counting

Having exhausted the possibilities of available equipment, we decided on what we wanted the ideal piece of equipment to do for us, and set about designing it with the intention of fabricating it ourselves. This proved to be a considerable task, as neither of us had any formal electronics design training and only basic electronics knowledge. Despite this we continued, and a basic flow schematic of our design is shown below in figure 6. See appendix A for the full schematic circuit diagram.

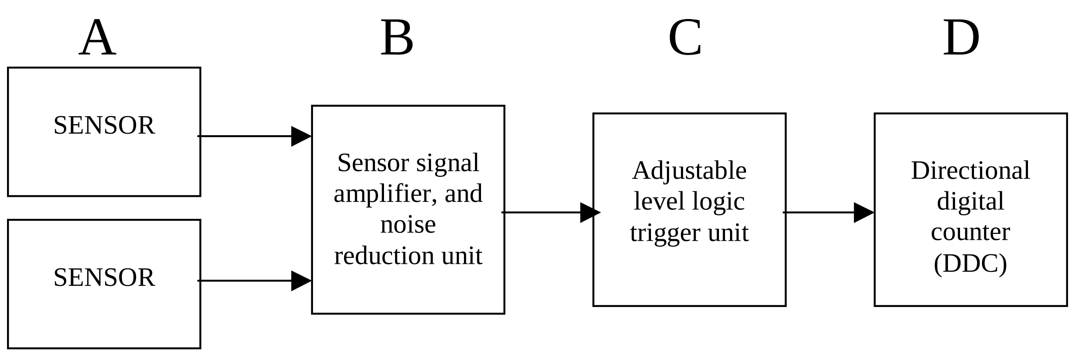

Figure 6. New hardware flow schematic.

Section A now holds two photodiode sensors, which allows for the measurement of the number of fringes moving in either direction. Section B centres on a high precision operational amplifier giving the input into section C an amplified signal, with the minimum noise. Section C consists of four Schmidt logic triggers. These allow us to specify two levels for the input signal from each sensor to be compared with. The first level is set just above the signal level of a dark fringe, with the second level set equally below the signal level of a bright fringe. The unit only gives a high signal out if the input signal has risen from below the first level to up above the second level. In effect we use the low setting of the first level, to act as a reset for the counting level set much higher. In this way the problem of noise causing many hundreds of counts is designed out of the system. Section D houses the majority of the digital components and two liquid crystal display counter units. This section looks constantly at the output from the trigger unit, which indicates the number of fringes passing the sensors. However from the way in which the sensors trigger, the DDC can also know in which direction the fringes are moving, it then increments the counter display that represents movement in the given direction by the number of fringes.

3.4 Alternative Set-up.

Figure 7 in appendix B shows a photograph of the set up we eventually used for measuring temperature of heated water in a beaker as it cools. Figure 8 shows what’s going on inside the beaker. A coil of bare fibre of length 53cm long. The remainder of the fibres was all encased in rubber tubing. In addition the unimmersed parts of the sensing fibre and all of the reference fibre were encased in a foam material.

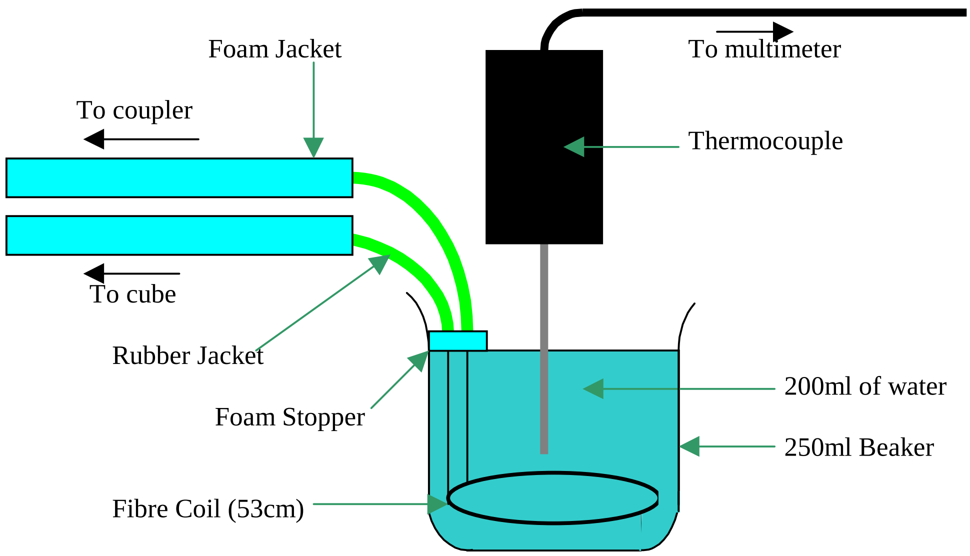

Figure 8. The beaker contents.

The foam stoppers (two strips of foam stuck onto the beaker with sellotape over the ends of the rubber tubing) were necessary to prevent water vapour from entering the rest of the fibres and warming them up. One might think that heat from the water could heat up the wrong bits of fibre by conduction, but that is not true. Let us suppose that the water is at 39°C, a point of the fibre 5cm away from the surface of the water is at 21°C and the room temperature is 19°C. Let us suppose that conduction has caused this situation. The conductivity of the core and cladding are approximately 1.35$Wm^{-1}K^{-1}$, [11]. The conductivity of the acrylate jacket is likely to be about 0.04$Wm^{-1}K^{-1}$, as is typical of plastics, [12]. The jacket is therefore likely to be still at room temperature. The average rate of heat conduction along this 5cm section of fibre is given is

$$\frac{dQ}{dt} = \frac{kA}{L}\Delta T \qquad (5)$$

where k is the conductivity, A is the cross sectional area of the core plus cladding ($1.23\times10^{-8}m^2$), [3], L is the length and ΔT is the difference in temperature (18°C). The answer is 9.94×10^{-7}W. There will also be conduction from this section of fibre through the jacket to the air. Using the same formula, taking L to be the thickness of the jacket, 60μm, A as the surface area of the section of fibre and ΔT as 2°C the conduction rate is 2.2W. This is in fact an underestimate because the temperature distribution along the fibre is likely to be exponential (i.e. ΔT should be larger) and there will be loses due to radiation. Conduction down the fibre is therefore negligible and this situation should never be reached in the first place. Figure 10 emphasises this. The timescale of these experiments shouldn't be something to worry about.

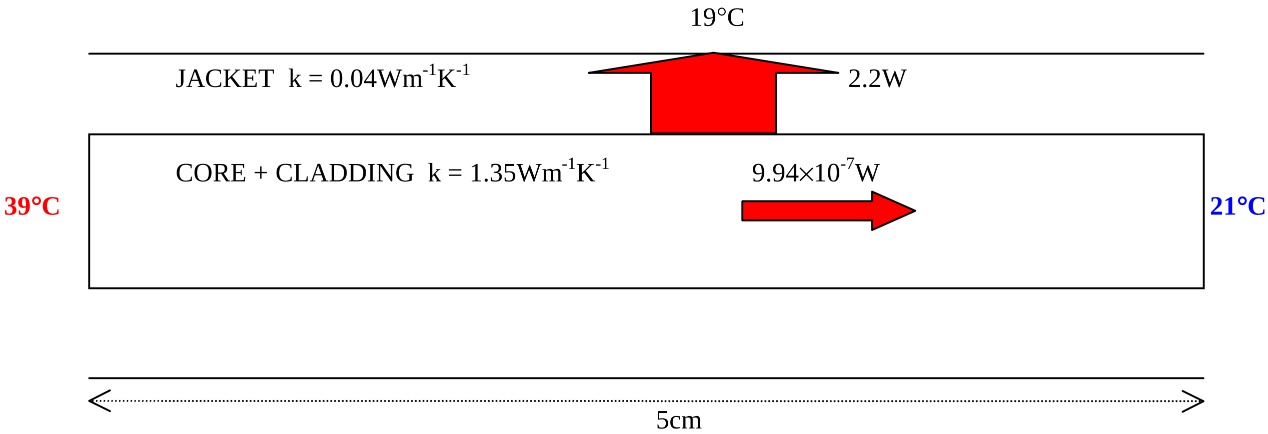

Figure 10. Conduction down the fibre is negligible.

The jackets cut down random fringe motions surprisingly well. At the start of each experimentation session it would take the interferometer about 20 minutes to adjust to the fact that something warm (i.e. a human being) had entered the room, and was busy moving around disturbing the normal temperature currents. After that the fringe pattern would stay remarkably still with one having to put one's fingers into the beaker to induce significant fringe motions. The result of this was that it was possible to stare at the pattern and count the fringes. An experiment was done. The water was allowed to cool from a temperature of 36.62°C to 33.62°C. The fringe count was 37 ± 1. This took about ten minutes. The number of fringes per °C change was 12.33 ± 0.30. Using equation for and the figures calculated the value should be 12.6, which is within our errors.

This shows that counting the fringes without electronic equipment is possible for a slow change provided that the interferometer fibre is insulated in the relevant places.

4. STRAIN AND PRESSURE

4.1 Strain

The core of the fibres was made of silica with traces of Germanium. According to Galiorenzi et al, [5], the Young's Modulus of this glass is 7.2×10^{10} Pa, that of the cladding is likely to be slightly smaller, but not much. With the set up shown in figure 4 the extension is given by

$$x^2 = \frac{mgL^2}{AY} \quad \text{if } x \ll L \qquad (6)$$

where m is the mass, Y is the Youngs Modulus, A is the cross sectional area, g is the gravitational field strength and L is the length of the fibre between the two supports. The Youngs modulus of the jacket is likely to be much lower than that of the glass so we can take $Y = 7 \times 10^{10}$ Pa and A equal to $1.23\times10^{-8}m^2$. If L= 16.6cm the mass required to produce a path difference of 1 wavelength turns out to be 0.6μg. In theory this kind of mass should be measurable.

From a piece of cardboard a shape was cut out which resembled a shepherd's crook. It had a hook such that it could be hung onto the suspended sensing fibre. A piece of foam was attached to inner edge of the hook to prevent it from damaging the fibre. The mass of the cardboard plus the foam was 0.758g as measured with a balance. The crook was 15.9cm long. It was given that length such that one's hand could kept at reasonable distance from the fibre when loading it on. Prior to loading the fringe pattern was reasonably still. The weight of the crook deflected the fibre by 5°±0.5° and thus the strain was 0.49mm ± 0.049mm. If our estimates our correct the deflection should have been 4.3°, which is not far away. The fibre returned to its original position once the weight had been removed, which was encouraging. Unfortunately the fringes moved at a rate which was impossible to count, in theory about 1000 in less than a second. WE have yet to find out whether the electronic equipment would cope.

4.2 Pressure

Let us suppose that a uniform pressure is applied uniformly around the surface area of a 30cm-length section of the fibre core. The change of length that results is given by this formula, [5],

$$\Delta L = \frac{2\nu PL}{Y} \qquad (7)$$

where ν is the Poisson ratio, P is the pressure and L is 30cm. The Poisson ratio of pure fused silica is 0.165, [11]. The pressure required to produce a path difference of one wavelength is 3.14×10^5 Pa which is approximately 7 atmospheres. This type of interferometer is therefore much less sensitive to pressure than it is to temperature or strain. Applying this kind of pressure uniformly around the fibre is not straightforward and thus it was not attempted.

CONCLUSION

Looking at the experiment as a whole, and the directions in which students have gone in the past, it appears as if we have achieved very little. However we have (with little electronics knowledge) built and debugged the majority of our new hardware, which functions satisfactorily we expect to complete the project and hope to have a finished model by the start of the next semester's laboratory schedule. We have achieved a good in depth knowledge of the electronics involved in our hardware, as well as the workings of the interferometer and the accompanying physics. One element that we now wish we could alter in the new hardware is that because of the nature of the sensor set up it can only count an integer number of fringe movements. Therefore it would be excellent for rapid temperature change or a shock load pressure change when the fringes would move many hundreds of places in a very short time. For longer duration gradual temperature or pressure changes however mean that the equipment could still have a significant error if the number of total fringes moved is not greater than 20-30 fringes. This would represent an optical length change of 12-19 microns in the fibre.

We could improve the hardware by redesigning the sensor array to utilise more sensors, that way we could at least count half fringes. We could use a computer to analyse the photodiode output, this would give us a much greater accuracy than we have now, but unfortunately a computer was not one of the available resources. According to Wilson and Hawkes [5] it should be possible to measure temperature changes down to 10^{-8} K, with shot noise then being the limiting factor. In our case the limiting factor was the ability of the counter to distinguish a single fringe movement, limiting the minimum detectable temperature change to approximately 0.1 K.

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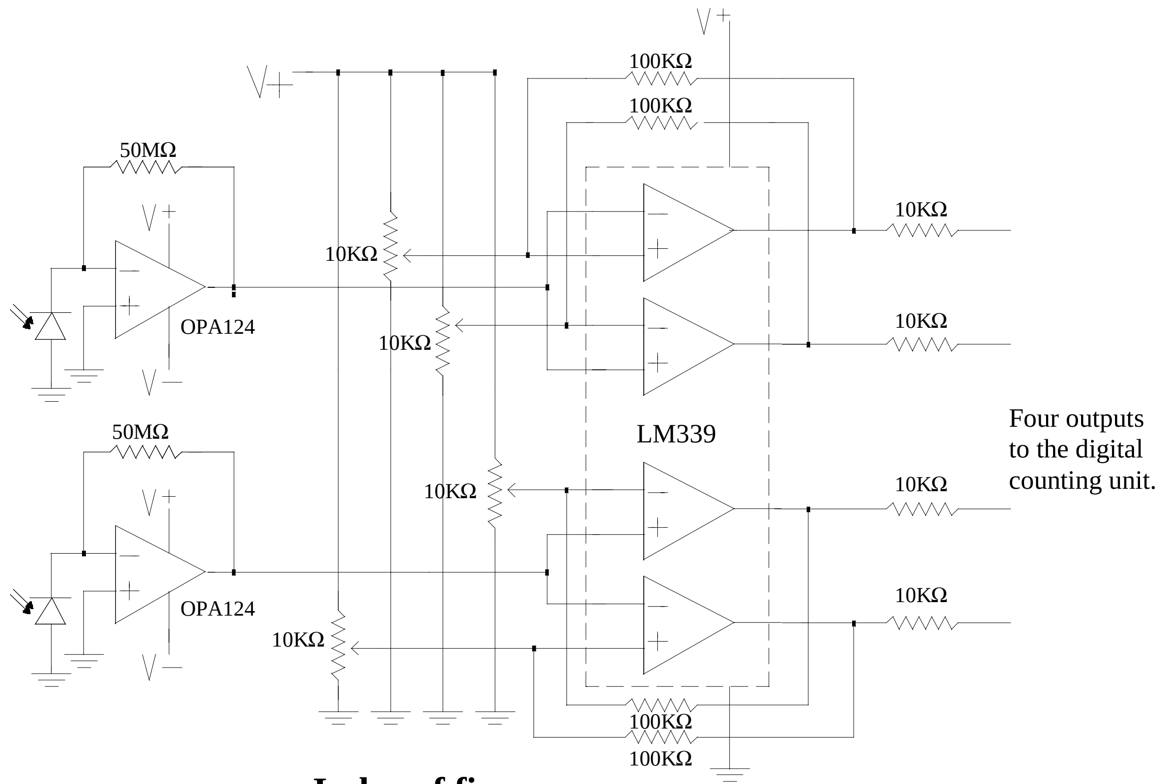
Appendix A **The completed cicuit so far.**



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