THE DETECTION OF GRAVITATIONAL WAVES

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The designing of a gravitational wave detector is an important excit6ing area of modern astronomy. However nobody has yet undisputedly detected any gravitational waves, let alone identified the source. In this essay I shall start by describing gravitational waves and some possible sources. I shall then go on to talk about some existing detectors. In the final part of this essay I shall promote the idea of using an elastic spherical mass as a detector.

1. THE BASIC PROPERTIES OF GRAVIATIONAL WAVES

Albert Einstein’s general theory of relativity predicted the existence of gravitational radiation. Such radiation should be analogous with electromagnetic radiation in that an accelerating mass should emit gravitational radiation in the same way that accelerating charge emits electromagnetic radiation. This radiation travels at the same speed, 2996792458m/s. Whereas it is possible to detect electromagnetic dipole radiation with a simple antenna it is not possible to detect gravitational dipole radiation. This is because of the equivalence principle that says that the inertial mass of an object is equal to its gravitational mass, [1]. However gravitational quadrupole radiation is, at least in theory, detectable. Figure 1 shows how a circle of point masses is distorted when a “sinusoidal” pulse of radiation passes perpendicularly through the centre of it. The circle is first deformed into an ellipse of the same area, then back into a circle, then into another ellipse in the opposite sense and so on. Notice that quadrupole radiation has two orthogonal polarisations. They are commonly labelled + (plus) and × (cross).

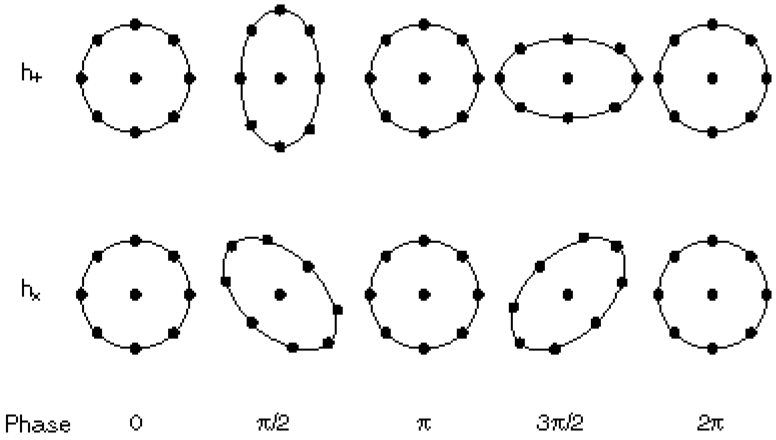
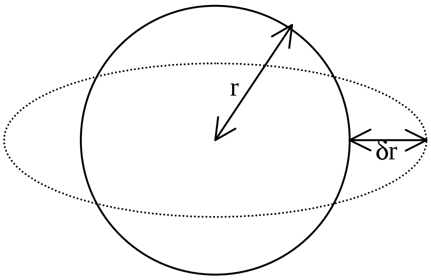


Figure 1. A circle on point masses being deformed by a gravitational wave coming out of the page. Time goes from left to right.

The amplitude of a gravitational wave is commonly defined by the dimensionless strain, h. Figure 2 show how h is defined.



$$h = \frac{\delta r}{r}$$

Figure 2.The amplitude of a gravitational wave is defined by how it affects a circle of masses

2. SOURCES OF GRAVITATIONAL WAVES

The quadrupole moment of an object or system of objects is given by this formula, [1].

$$Q = \iiint \left(3z^2 - r^2\right)\rho\left(x, y, z\right) dxdydz$$

If the third differential of the quadrupole moment is non- zero it will emit gravitational radiation. Nothing on earth can produce gravitational radiation that is nearly detectable. The radiation we receive from astronomical sources is much stronger however. In 1974 a binary neutron star was discovered in the constellation of Aquila . It is 5000 parsecs away. It is named PSR 1913 +16. The two neutron stars are separated by only about 1.5 million kilometres on average and each orbits its common centre of mass in however this period is increasing by 75μs per year. At the moment it is roughly 0.0608s. This rate of increase in period is consistent with the idea that the system is losing energy by means of emitting gravitational radiation, [2]. This means that although no radiation has been actually detected there is very good evidence that it exists. Einstein's prediction is therefore correct and we are justified in searching for radiation. A binary pulsar is an example of a periodic source of gravitational waves. The period of the radiation emitted by a binary star system is equal to half the period of revolution of the system, [3], and thus is 32.9Hz for PSR 1913 +16.

An example of a burst source of gravitational waves source is a supernova. A star, during its collapse and subsequent outburst remains approximately spherical, but is not completely so and it takes place very quickly. It has been estimated that a type II supernova in our galaxy might produce waves with a frequency of 1kHz and a maximum h-value of 10^{-18}. This highlights the problem. Two masses separated by 1m on Earth are move apart through a distance that is very much smaller than the diameter of a proton. In spite of that many existing detectors are sensitive enough to detect this but a galactic supernova occurs only once every 30 years. Our aim is to extend our range to the Virgo supercluster of galaxies, whose centre is about 15Mpc away, [1]. Other possible sources might be supergiant black holes swallowing stars, starquakes from neutron stars or, more speculatively, the coalescing of MACHOS in the galactic halo, [4].

In addition to measuring radiation from the types two of source mentioned we are hoping to measure the stochastic or background emission. This may be due to activities in the early Universe are a superposition of weak sources such as ordinary binary stars, [5].

3. BAR DETECTORS AND INTERFEROMETERS

In 1959 J. Weber came up with the idea of using a large piezoelectric quartz crystal as a gravitational wave detector, [6]. In 1966 he built the first detector. It was an aluminium cylinder of length 1m and diameter 60cm. Figure 2 shows how a metal bar is distorted by a gravitational waves.

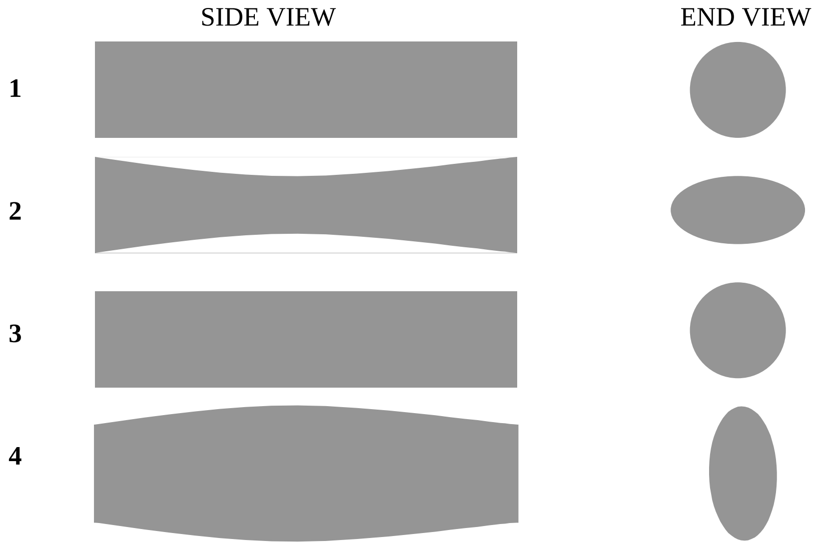


Figure 3. An aluminium bar starts vibrating when a gravitational wave passes through it longitudinally. This motion is, of course, exaggerated and idealised.

He cooled it down to 20K and used piezoelectric transducers (PZTs) to convert small-scale motions on the surface of the bar into an electrical signal. PZTs are usually made using the ceramic lead zirconate titanate A sensitivity of 1 part in 10^{16} was reached.

Since then other bar detectors have been developed in the likes of ALLEGRO, EXPLORER and NAUTILUS. The latter two have been developed at Laborati Nationali di Frascati in Rome. They are very similar detectors. The bars are both made of aluminium 5056, they weigh 2350kg and are both 3m long and 60cm in diameter. Their fundamental frequencies of vibration are 908Hz and 924Hz (and so are best suited for detecting burst sources), [4]. The strong similarities are important because it gives us the possibility of measuring the radiation of the same source by two separate detectors in coincidence. NAUTILUS uses a ^{3}He / ^{4}He dilution refrigerator. To keep it at 0.1K 90 litres of superfluid helium are required per day, [4]. At this temperature. For a bar detector the noise level h-value is given approximately by this formula, [4],

$$h_{\min}(f_0) \approx \frac{2L}{v_s^{\;2}}\sqrt{\frac{kT_{\mathrm{eff}}}{M}}$$

where M is the bar's mass, v_s is the sound speed, L is the bar's length, k is Boltzmann's constant and T_{eff} is

$$T_{\mathrm{eff}} = 4T\sqrt{\frac{T_n}{\beta Q(T+T_b)}}$$

where T is the thermodynamic temperature, T_n is the SQUID (amplifier noise temperature, T_b is the back-action noise temperature, Q is the mechanical quality factor, β is the energy coupling factor between the bar and PZTs. All bar detectors use aluminium alloys because they have high mechanical quality factors at low temperatures (sharp resonance peaks) and have high sound speeds. For NAUTILUS the Q-values have been calculated to be 1.26×10^6 for the lower frequency and 2.3×10^6 for the higher one. The coupling factors have found to be 3.5×10^{-4} and 3.9×10^{-4} The measured noise level at the resonant frequencies is equivalent to a burst of gravitational waves of amplitude $h = 5 \times 10^{-19}$

The theoretically lowest possible amplifier noise temperature for a bar detector is of the order of 0.1μK, as is defined by the zero-point energy ($hf \geq kT$). If we could its noise temperature down to the single phonon limit it might be possible to measure strains of 3×10^{-21}. Radiation from supernovae in the Virgo supercluster might then be observable.

The other type of detector that has been developed is the interferometer type. The most famous example is LIGO (Laser Interferometer Gravitational wave Observatory). It has three sites, one in Washington, one in Louisiana and one in Los Angeles. Figure 4 shows the principle of how it works. The coherent monochromatic light from a laser is split by a beam splitter and then diverted towards a pair of suspended mirrors. The L.A. site has two end stations 4km away from the corner station. At the other two sites there are two end stations and two mid-stations which are 2km away from the corner station and also contain mirrors. The arm length at these stations can be set to either 2km or 4km, [6]. When the beam hits the mirror its reflected back to a mirror at the corner station and then back again 400 times over before arriving back at the beam splitter. The effective path length is therefore 3200km (or 1600km). If the two beam arrive back at the beam-splitter in phase the photodiode will not do anything. The affect of a gravitational wave is to shorten one arm and lengthen the other. When that happens the two beams do not arrive back in phase and a voltage is induced in the photodiode.

Powerful measures are needed to cut down noise as much as possible. For example the pressure inside the 8100m^3 system has to be less than 10^{-12} atmospheres as to cut down scattering as much as possible, [6]. However the ability of this type of detector is ultimately limited by the fact that if N photons are expected to arrive at a certain place at a certain time the standard deviation of that number is the square root of N. And consequently, if the arm-length is 400km and the laser wavelength is 500nm, [7],

$$h_{\min} \approx \left(\frac{1}{\sqrt{2\times 10^{20}}}\right)\left(\frac{500\mathrm{nm}}{4\mathrm{km}\times 400}\right) \approx 10^{-23}.$$

Although they are very expensive to run and set-up they do have some potential.

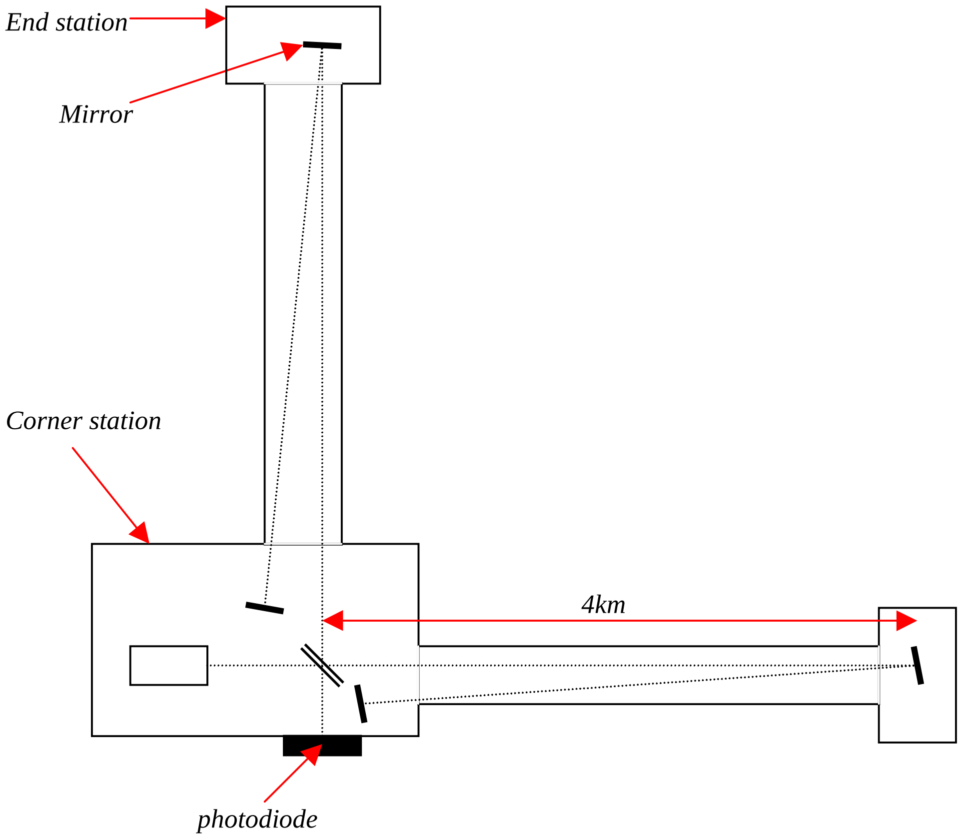


Figure 5. A schematic diagram of Los Angeles LIGO site.

Bar detectors are best suited for detecting radiation of 800Hz to 1000Hz. LIGO is better suited for detecting radiation between 10Hz and 100Hz. It turns out that it is virtually impossible too detect any radiation below 1Hz because the seismic noise is too high, no matter how well suspended the masses or mirrors are. There are plans to send an interferometer known as LISA (Laser Interferometer Space Antenna) into outer space. It will have an arm length of 5 million km. Its aim to detect radiation between 10^{-4}Hz and 0.1Hz.

4. A SPHERICAL GRAVITATIONAL WAVE ANTENNA

The bar detector and the interferometer detectors share one common problem. Unless the wave fronts arrive at the detector perpendicularly to the plane of the interferometer of the round face of the bar not all of the information will be recorded. Identifying a signal with a source is therefore not straightforward. On the other hand a sphere has an isotropic cross-section (it is the same when viewed from any direction).

So let us crudely design a resonant spherical mass detector. Let it be made of aluminium. It is going to be placed a vacuum chamber so let be oxygen free. This means that the edges of the block from which the sphere is cut must be excluded, as they will be slightly oxidised. Once it has been cut it should be placed inside a temporary vacuum chamber (e.g. a bell jar). It is perhaps worth saying at this point that the accuracy to within which a spherical detector needs to be cut is only about 1%. In the case of an interferometer everything needs to be positioned correctly to within one atom! Also the oxygen problem is more acute in an interferometer detector like LIGO as the walls of the vacuum tubes are made of a special kind of steel that has a very low dissolved hydrogen content, [7]. Any resonant mass detector must be suspended on a wire. This is the only way of effectively cutting out seismic vibrations. NAUTILUS used oxygen free high conductivity cable (OFHC), so we will use that. In NAUTILUS' case an overall vibration isolation of –260dB was achieved at the bar's resonant frequencies, [4]. This is equivalent to a factor of 10^{-24}. The cable also has a second function, it provides a thermal link between the antenna and the refrigerator, [4].

If a metal sphere is suspended on a thin rod and struck with a hammer it will vibrate with very many modes at very many frequencies. A gravitational wave however is not like a hammer and only couples with the spheroidal quadrupole modes of vibration of the sphere. The tidal force density applied by a gravitational wave to matter is given by this formula, [5].

$$F_i(\underline{x},t) = \frac{1}{2}\rho\sum_j \frac{\partial h_{ij}(t)}{\partial t^2} x_j$$

where h_{ij} is a strain tensor. If the z-axis is chosen to be parallel with the direction of the incoming wave it is as below.

$$h_{ij} = \begin{bmatrix} h_+(t) & h_\times(t) & 0 \\ h_\times(t) & -h_+(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The tidal force can be expressed as the gradient of a scalar field. This scalar field can be shown to be

$$\Phi(\underline{x},t) = \sqrt{\frac{\pi}{15}}\rho r^2 \sum_{n=1}^{5} \ddot{h}_n(t) Y_n$$

where Y_n is a special set of linear combinations of the l=2 spherical harmonics, [9] they are

$$\begin{aligned}
Y_1 &= \sqrt{\tfrac{1}{2}}\left(Y_{2,2} + Y_{2,-2}\right) = \sqrt{\tfrac{45}{48\pi}}\left(1-\cos^2\theta\right)\cos(2\phi) \\
Y_2 &= i\sqrt{\tfrac{1}{2}}\left(Y_{2,2} + Y_{2,-2}\right) = \sqrt{\tfrac{45}{48\pi}}\left(1-\cos^2\theta\right)\sin(2\phi) \\
Y_3 &= \sqrt{\tfrac{1}{2}}\left(Y_{2,1} + Y_{2,-1}\right) = \sqrt{\tfrac{45}{12\pi}}\cos\theta\sqrt{1-\cos^2\theta}\cos\phi \\
Y_4 &= i\sqrt{\tfrac{1}{2}}\left(Y_{2,1} + Y_{2,-1}\right) = \sqrt{\tfrac{45}{12\pi}}\cos\theta\sqrt{1-\cos^2\theta}\sin\phi \\
Y_5 &= Y_{2,0} = \sqrt{\tfrac{5}{16\pi}}\left(1-\cos^2\theta\right) \quad .
\end{aligned}$$

The angular dependence of the radial motion of five quadrupole modes of vibration of the sphere are defined by these spherical harmonics. Each of these modes are degenerate. Figure 5 shows 5 spheres; each one is vibrating with one of the 5 modes. If, when a wave hits the antenna, we can decipher which modes are excited and to what degree they are excited, we can work out the amplitude, the polarisation and the direction of propagation of the wave. The radial dependence on the motion of the sphere is a linear combination of Bessel functions of the first and second kinds. The motion of the sphere's surface is not purely radial; there is a small amount of tangential motion as well.

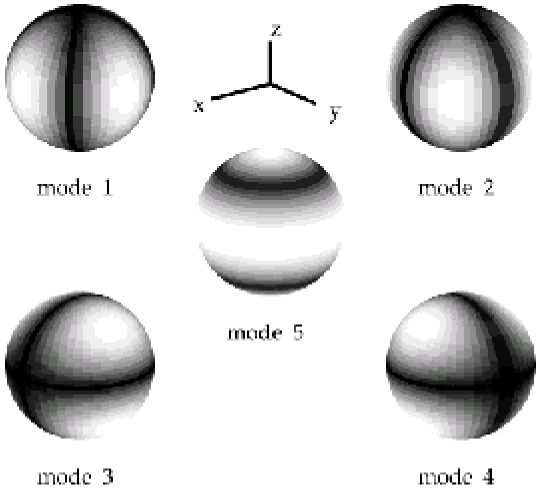


Figure 5. For each mode the lighter areas show where the radial motion is the greatest (i.e. antinodes) and the darker areas are where the radial motion is the smaller or zero (i.e. nodes), [5].

The lowest quadrupole frequency of a sphere is, in fact, the lowest of all its vibration frequencies, [10]. This means that once excited, it will continue to vibrate for longer at that frequency than any of its others. Its value can be shown to be expressed by this formula

$$\frac{\omega R}{v_s} = f(\sigma)$$

where R is the sphere's radius, ω is the angular frequency, v_s is the material's sound speed and $f(\sigma)$ is a weak function of the material's Poisson ratio, [10]. Aluminium has a Poisson ratio of 0.35 at low temperatures. If we chose the fundamental frequency of our sphere to be 1000Hz, its radius would have to be 1.29m and its weight would be 24278kg. Sadly one of the drawbacks of a spherical detector is that for a given frequency it has to be ten times heavier than the equivalent bar detector. On the other hand it's energy cross-section is 56 times better than that of a bar detector's averaged over all angles and about 9 times better than a bar detector's if the wave comes in head-on, [11]. The fore-mentioned antenna mass would produce a 5% strain in a copper cable that had a cross-sectional area of 4mm^2. A thin cable attenuates better than a thick cable but it might be dangerous to go beyond a 5% strain. The length of the cable excluding any part inside the antenna should no more than 25cm. With that length the fundamental frequency of vibration of the cable is about 10kHz and its other frequencies. That frequency should be sufficiently far away from the antenna's resonance peak.

The next problem is how to measure the vibrations. In 1994 Zhou and Michelson, [11], came up with following idea. If an incoming wave is parallel to the z-axis, there should be transducers at the following 5 positions: $\theta = \pi/2$, $\phi = \pi/4$ because there is no azimuthal motion in any of the modes except mode 1; $\theta = \pi/2$, $\phi = \pi$ because only mode 2 has azimuthal motion there; $\theta = \pi/2$, $\phi = 0$ because only mode 3 has any motion in the θ-direction; $\theta = \pi/2$, $\phi = \pi/2$ because only mode 4 has any θ-direction motion and $\theta = 0$, $\phi = 0$ because only mode 5 has any radial motion there. This means having 1 PZT measuring radial motion and 4 measuring tangential motion. This arrangement has its drawbacks. The tangential motions are very small indeed, the sphere plus resonator system is not spherically symmetric and is of little use if the wave comes in at an angle which is way off the z-axis. In 1995 Merkowitz and Johnson, [5], [12], proposed an alternative arrangement using piezoelectric resonators. Figure 6 are an alternative to ordinary PZTs for measuring radial motions. It is effectively a small mass on the end a piezoelectric spring, which resonates at the frequency we want. It's mass is much smaller than the antenna's but not negligible. A prototype truncated icosahedron was cut out a bar of aluminium 6063, which was formerly a gravitational wave antenna! The TIGA's (Truncated Icosahedron Gravitational wave Antenna's) resonant frequencies are close to 3235Hz[*]. This frequency is probably too high for it to ever make any observations but has been used as a prototype spherical detector in impulse excitation experiments. Figure 7 shows the above view of a truncated icosahedron with 6 resonators positioned in the arrangement specified by Merkowitz and Johnson. The resonator postions are

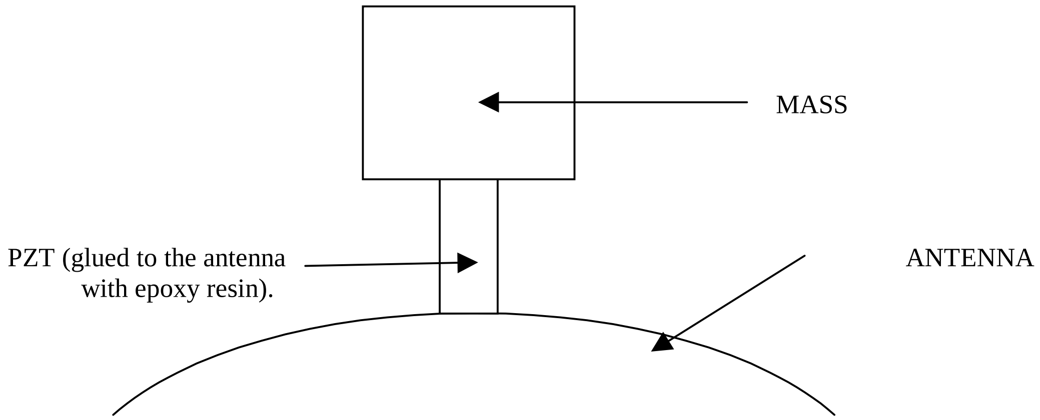


Figure 6. A piezoelectric resonator, exaggerated in size

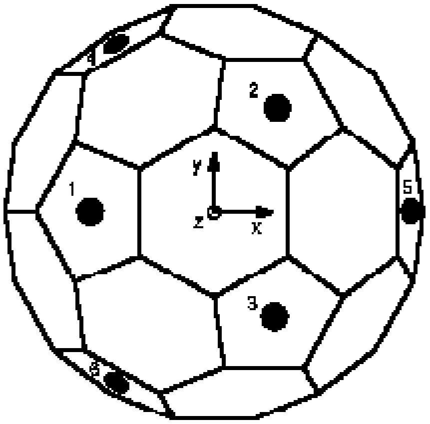


Figure 7. A truncated icosahedron (TI) with 6 resonators positioned in the truncated icosahedral arrangemen, [5].

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[*] Its 5 quadrupole modes are not degenerate because it is only an approximation to a sphere and it has a suspension hole passing right through its centre.

shown in table 1. They happen to be centres of the centres of the pentagon faces on the TI.

TABLE 1

RESONATOR NUMBER	θ / °	ϕ / °
1	37.3773	180
2	37.3773	60
3	37.3773	300
4	79.1876	120
5	79.1876	0
6	79.1876	240

Because of the symmetry of the system and the vibration modes it is not necessary to have resonators on the opposite faces. There is however no harm in doing so and it preserves the symmetry of the antenna to some extent. Although this arrangement was originally tried on a TI, it can be used any spherical detector, and is still known as the truncated icosahedral arrangement. It is possible, using special electronic equipment, to form fixed linear combinations of the resonator outputs, known as mode channels, to form mode amplitudes. Figure 8 shows what the resonator outputs ought to look like when an $h_\times$ polarised gravitational wave travels down the z-axis and hits the antenna. Figure 9 shows the result we'd like to get.

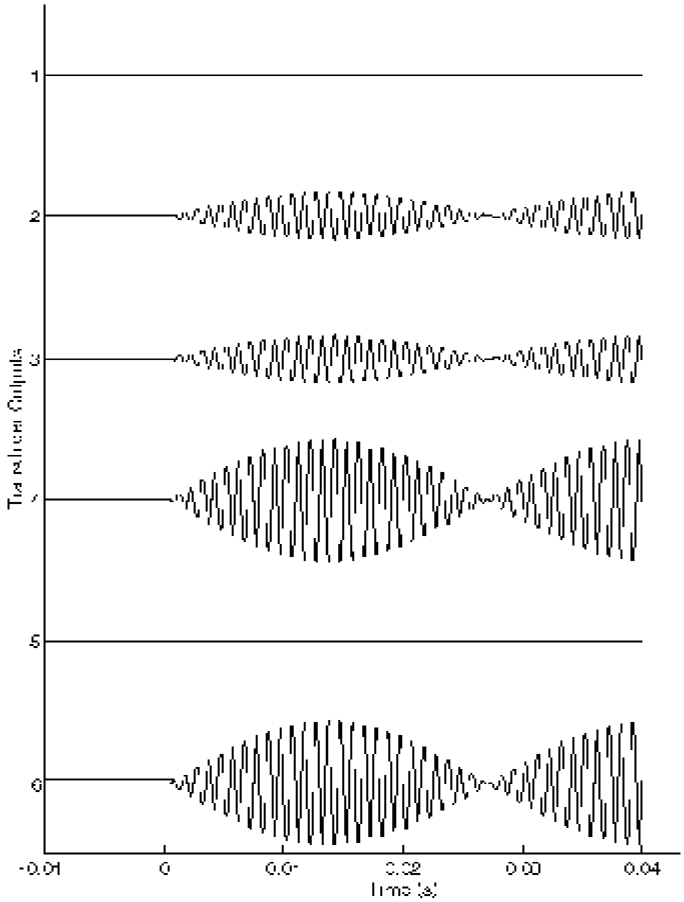

Figure 8. What the resonator outputs might look like if an hx gravitational wave burst parallel to the z-axis were to hit the antenna, [5].

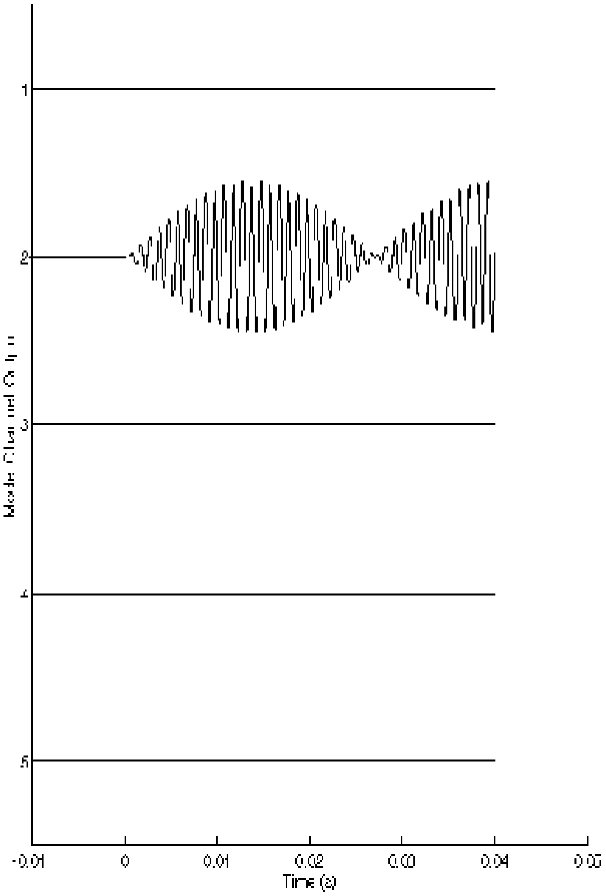

Figure 9. The only excited mode is mode 2, [5].

Figure 10 shows the detector design do far with a few additions

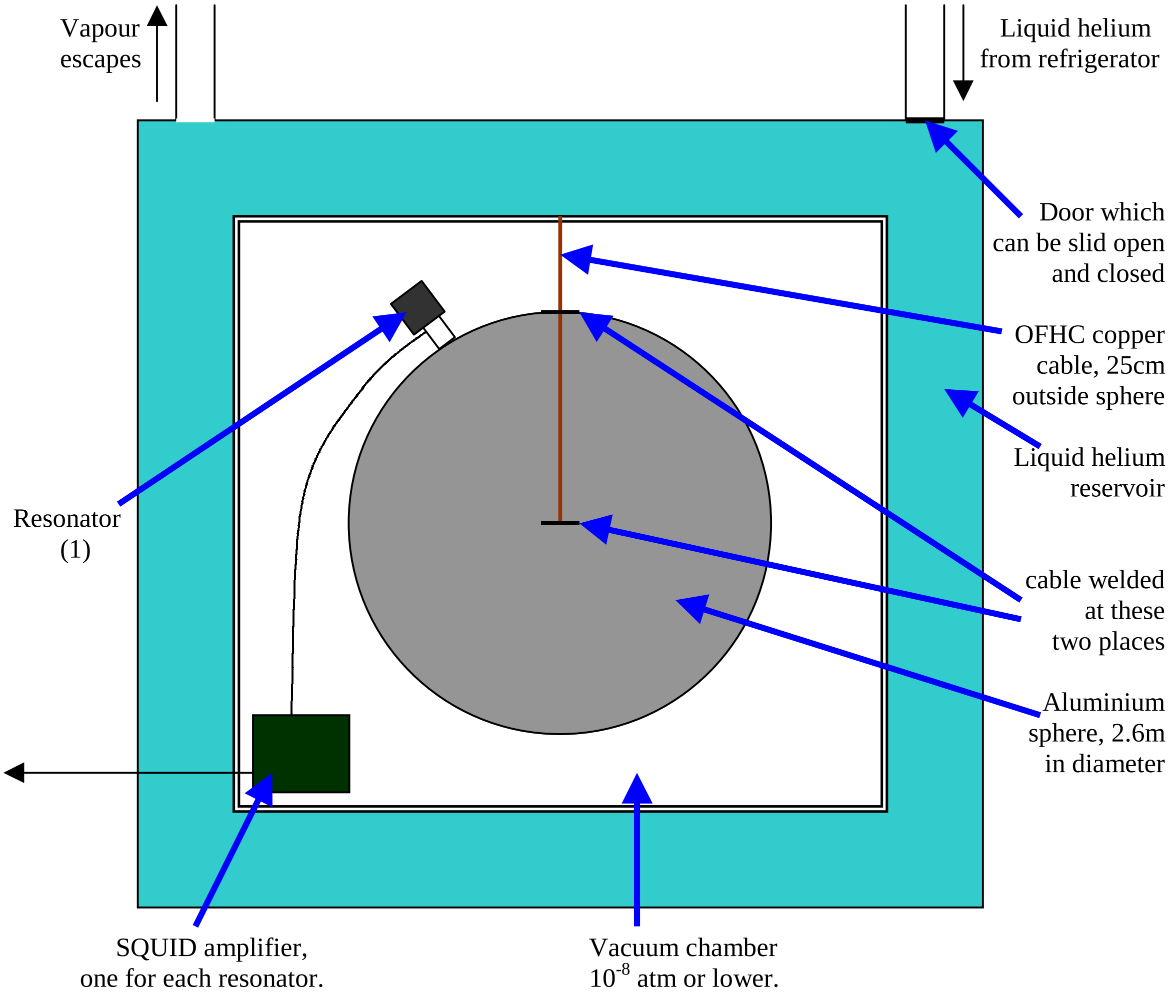

Figure 10. The basic set-up of the central area of the detector Only one of the 12 resonators and its SQUID amplifier is shown. If the x-axis points to the right and the z-axis is vertical then the resonator shown is resonator 1.

The vacuum chamber would probably have to be cube whose sides were at least 4m. The pressure must 10^{-8} atmospheres or lower to keep the residual air in gaseous form. Since NAUTILUS is the most promising bar detector to date it seems to make sense to keep the cryogenic apparatus more or less the same. The resonator outputs should be fed into SQUID amplifiers, which are inside the vacuum chamber. There should be a cryostat in the liquid helium chamber. The helium is allowed to evaporate but not allowed to boil because that would be another source of seismic noise, [4]. NAUTILUS is equipped with a cosmic ray veto system. It has particle detectors around its perimeter as to detect any cosmic air showers and also energetic single particles like muons or hadrons. These will become important as the single phonon limit is approached because these particles might interact with the antenna could simulate the effect of a burst of gravitational waves. Such detectors should enable us to correlate against such events. Encasing the detector in a shield of lead is probably not a practical alternative.

A XYLOPHONE OF SPEHRES

The above design is not going to build or run. If the funds were available though one way of extending this design would be to a line of spheres which are each sensitive to gravitational waves of lower frequency. One way of doing this is to increase the size of the sphere. A better way would be to use hollow spheres. One disadvantage of using a hollow sphere is that its energy cross section at its fundamental for a solid sphere of the same volume is much smaller, or indeed one of the same mass [13]. However the reverse is true regarding the second lowest frequency. Coccia et al., [13] have calculated the lowest two quadrupole frequencies and the respective cross-sections. Some of them are listed in table 2 below.

Table 2

ALLOY	CuAl	CuAl	Al 5056	Al 5056
Mass / tons	200	200	200	100
Diameter / m	4	6	6	6
Thickness / cm	81	25	90	37
Lowest quadrupole frequency / Hz	395	191	273	230
2nd lowest quadrupole frequency / Hz	1188	753	935	896
σ at lowest frequency / $10^{-23}m^2Hz$	1.5	1.1	1.8	0.8
σ at 2nd lowest frequency / $10^{-23}m^2Hz$	2.4	1.7	2.9	1.2

For my design I therefore propose a "xylophone" of 6 spheres.

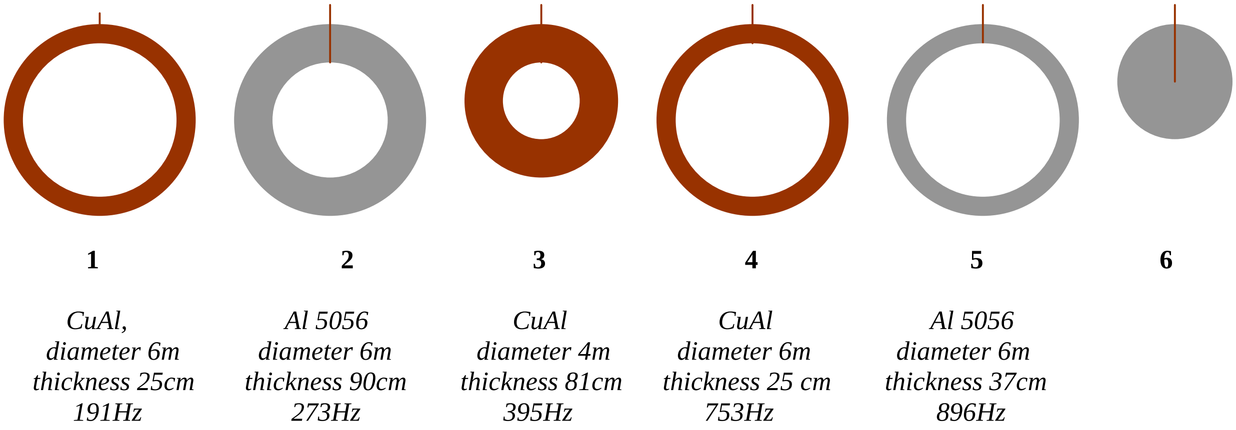

Figure 11. A xylophone of 6 spheres. Sphere 6 is the original solid sphere. The frequencies stated are the frequencies to which the attached resonators should be tuned.

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